PAPER: DISCRETE MATHEMATICS

SPECIALITY: COMPUTER **ENGINEERING**
OPTION: **DBM**
EXAM PAPER: **MATHEMATICS AND STATISTIC**
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SECTION A MCQ **20 marks**

1. A parameter is:

    a. a sample characteristic
    b. a population characteristic
    c. unknown
    d. normal normally distributed

2. A statistic is:

    a. a sample characteristic
    b. a population characteristic
    c. unknown
    d. normally distributed

3. A national random sample of 20 ACT scores from 2010 is listed below. Calculate the sample mean and standard deviation.

    29, 26, 13, 23, 23, 25, 17, 22, 17, 19, 12, 26, 30, 30, 18, 14, 12, 26, 17, 18

    a. 20.50, 5.79
    b. 20.50, 5.94
    c. 20.85, 5.79
    d. 20.85, 5.94

4. Provided that the ACT is reasonably normally distributed with a mean of 18 and standard deviation of 6, determine the proportion of students with a 33 or higher.

    a. 0.0062
    b. 0.0109
    c. 0.0124

d. 0.0217

5. Using the data in question 3, calculate the 95% confidence interval for the mean ACT score based on the *t*-distribution.

   a. -∞ to 23.05
   b. -∞ to 23.15
   c. 18.07 to 23.63
   d. 18.22 to 23.48

6. Using the data in question 3, calculate number of observations that are two or more sample standard deviations from the sample mean.

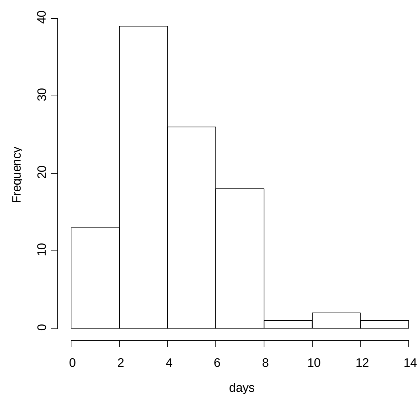
   a. 0
   b. 1
   c. 2
   d. 3

7. When asked questions concerning personal hygiene, people commonly lie. This is an example of:

   a. sampling bias
   b. confounding
   c. non-response bias
   d. response bias

8. Select the order of sampling schemes from best to worst.

   a. simple random, stratified, convenience
   b. simple random, convenience, stratified
   c. stratified, simple random, convenience
   d. stratified, convenience, simple random



9. The histogram above represents the lifespan of a random sample of a particular type of insect. Determine the relationship between the mean and median.
   a. mean = median
   b. mean ≈ median
   c. mean < median
   d. mean > median

10. When the correlation coefficient, *r*, is close to one:
   a. there is no relationship between the two variables
   b. there is a strong linear relationship between the two variables
   c. it is impossible to tell if there is a relationship between the two variables
   d. the slope of the regression line will be close to one

11. A random variable is
   A. Hypothetical list of possible outcomes of a random phenomenon.
   B. Any phenomenon in which outcomes are equally likely.
   C. Any number that changes in a predictable way in the long run.
   D. A variable whose value is a numerical outcome of a random phenomenon.

12. Workplace accidents are categorized in three groups: minor, moderate and severe. The probability that a given accident is minor is 0.5, that it is moderate is 0.4, and that it is severe is 0.1. Two accidents occur independently in one month. Calculate the probability that neither accident is severe and atmost one is moderate.
A. 0.25 B. 0.40 C. 0.45 D. 0.65

13. Let X1, X2, X3 be uniform random variables on the interval (0, 1) with Cov(Xi, Xj) = 1/24 for i, j =1, 2, 3, i≠j. Calculate the variance of X1 + 2X2 − X3.
A. 1/6 B. 1/4 C. 5/12 D. ½

14. A class contains 8 boys and 7 girls. The teacher selects 3 of the children at random and without replacement. Calculate the probability that the number of boys selected exceeds the number of girls selected.
A. 337/5512 B. 28/65 C. 8/15 D. 36/65

15. Let X and Y be discrete random variables with joint probability function p(x, y) given by the following table :

	$x = 2$	$x = 3$	$x = 4$	$x = 5$
$y = 0$	0.05	0.05	0.15	0.05
$y = 1$	0.40	0	0	0
$y = 2$	0.05	0.15	0.10	0

For this joint distribution E(X) = 2.85 and E(Y) = 1. Calculate Cov(X, Y).
A. − 0.20 B. − 0.15 C. 0.95 D. 2.70

16. Three factories A, B, C have 100, 200 and 300 workers respectively. The mean of the wages is the same in the three factories. Which of the following statements is true?

A. There is greater variation in factory C.
B. Standard deviation in. factory A is the smallest.
C. Standard deviation in all the three factories are equal
D. None of the above

17. In a discrete probability distribution, the sum of all the probabilities is always equal *to:*

A. Zero
B. One
C. Minimum
D. Maximum

18. What is the median of the sample 5, 5, 11, 9, 8, 5, 8?

a. 5
b. 6
c. 8

d. 9

19. Two events each have probability 0.2 of occurring and are independent. The probability that neither occur is

   a. 0.64
   b. 0.04
   c. 0.2
   d. 0.4

20. A smoke-detector system consists of two parts *A* and *B*. If smoke occurs then the item *A* detects it with probability 0.95, the item *B* detects it with probability 0.98 whereas both of them detect it with probability 0.94. What is the probability that the smoke will not be detected?

   a. 0.99
   b. 0.01
   c. 0.04
   d. 0.96

21. .A set A is said to be countable if there exists a function f:A→N such that

   (a) f is bijective

   (b) f is surjective

   (c) f is identity map

   (d) None of these.

22. Which of the following is not true for a set in **R**?

   (a) A set may not have an infimum in **R**.

   (b) Infimum of a set may not belong to the set.

   (c) Infimum and supremum of a set may be equal.

   (d) Supremum of a bounded below set always exists in **R**.

23. Which algebraic property is not true for the set of real numbers **R**?

   a) For all such that

   b) (1/ a) = 1 for all a .

   c) a^2 = a for all a G R.

   d) If a.b > 0 then either a > 0 and b > 0 or a < 0 and b < 0.

24. Which of the following is true?

   (a) Every decreasing sequence of real number is convergent.

   (b) Every monotone sequence is convergent.

   (c) Constant sequence is not convergent.

   (d) None of these.

26. The sequence

25. . Which of the following sets is countable?

(a) The set [0,1].

(b) The set R of all real numbers.

(c) The set Q of all rational numbers.

(d) None of these.

(a) is convergent.

(b) is divergent.

(c) is convergent to 0.

(d) is convergent to 1.

27. If the sequence is convergent then

(a) it has two limits.

(b) it is bounded.

(c) it is bounded above but may not be bounded below.

(d) it is bounded below but may not be bounded above.

28. If the sequence is increasing, then it

(a) converges to its supremum.

(b) diverge.

(c) may converge to its supremum.

(d) is bounded.

29. Which of the following is not true?

(a) The set [0,1] is a finite set.

(b) The set R of all real numbers is uncountable.

(c) The set Q of all rational numbers is countable.

(d) None of these

30. Every convergent sequence has …………….. one limit.

a. (A) at least
b. (B) at most
c. 2(C) exactly
d. (D) none of these

31. If $y = ax + c$ denotes the *slope-intersect* form of a line in 2D, then c gives us...

a) .the slope of the line
b) .the fraction of the slope in the x-direction
c) .the fraction of the slope in the y-direction
d) .the intersection of the line with the y-axis

32. If 2x — y + 5 = 0 denotes the *implicit* representation of a line in 2D, then the vector (2, -1) is.

a) .a point on the line
b) .a vector parallel to the line
c) .a vector perpendicular to the line
d) none of the above.

33. If *p(t)* = (1,1) + t(—2,1) denotes the *parametric* equation of a line in 2D, then (1,1) is.

a) .a point on the line
b) .a vector parallel to the line
c) .a vector perpendicular to the line
d) none of the above.

34. The equation $2x + y + z = 0$ represents.

a) .the implicit representation of a line in 3D
b) .the implicit representation of a plane in 3D
c) .the implicit representation of a line or a plane in 3D
d) none of the above.

35. In K^3, the equation $(x — 3)^2 + (y — 3)^2 + (z — 3)^2 - 9 = 0$ represents.

a) .the implicit representation of a sphere with radius 9 and center (3,3,3)
b) .the implicit representation of a sphere with radius 3 and center (3,3,3)
c) .the implicit representation of a sphere with radius 9 and center (—3,—3,—3)
d) .the implicit representation of a sphere with radius 3 and center (—3,—3,—3).

36. If the scalar product (dot product) of two unit vectors is zero, they are.

a) .linearly dependent
b) .forming an orthonormal basis
c) .pointing in the same direction
d) .at an angle of 180 degrees to each other.

37. Calculate the area of the parallelogram with the given vertices.
(0, 0), (2, 6), (11, 8), (9, 2)

a) 52 b) 100 c) 49 d) 50

38. Let A = (-5 2] and B = [1 0]. Find 2A+3B.

a) [-10 4] b) [-2 2] c)[-9 4] d) [-7 4]

39. Find the matrix product AB if it is defined by

a.
b.
c. AB is undefined
d.

40. Find the inverse of the matrix if it exists

a.
b.
c.
d.

SECTION B: STRUCTURALS

1. Analysis 30 marks

1.1.

a. Find the maclaurin expansion of $\tan^{-1}(x^2)$ up to and including the term in x^4.

b. Hence or otherwise, determine $\int_0^1 \tan^{-1}(x^2)$

1.2.Consider the function

$$f(x,y) = \begin{cases} 2\dfrac{x^3 y}{x^2 + 2y^2}\cos(xy) & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

a. Find the partial derivatives of f at the point $(0,0)$

b. Prove that f is continuous on all [?]. Hint: Note that for $(x, y) \neq (0,0)$ we have that $\frac{1}{x^2 + 2y^2} \leq \frac{1}{x^2 + y^2}$

c. Is f differentiable at (0,0) ?

1.3.Find the Fourier series of the function f defined by

$$f(x) = \begin{cases} -1 & \text{if} \quad -\pi < x < 0 \\ 1 & \text{if} \quad 0 < x < \pi \end{cases}$$

and f has period 2π. What does the Fourier series converge to at $x = 0$?

1.4.Let $K(x, y) = \ln\left(x^2 + y^2\right)$. For $(x, y) \neq (0,0)$,

a. Find the first and second partial derivatives of $K(\mathrm{x}, \mathrm{y})$.

b. show that $K_{xx} + K_{yy} = 0$.

c. Consider the function

$$f(x, y) = \sin(x + y) \quad \text{with} \quad x = st^2 \quad \text{and} \quad y = s^2 + 1/t$$

Compute f_s and f_t ;

i. Using the chain rule

ii. Using any other method

2. STATISTICS 25 marks

2.1.Suppose that a discrete random variable X has the following probability distribution.

X	1	3	5
P(X)	1/4	1/4	1/2

(a) Find the mean μ_X of X.
(b) Find the variance $(\sigma_X)^2$ of X.
(c) Define the new random variable Y = 3X + 1. Use the properties of the mean of linear functions of random variables and your results in the previous parts to find the mean of Y.
(d) Use the properties of the variance of linear functions of random variables to calculate the variance and standard deviation of the new random variable Y

3. PROBABILITY 25 marks

3.1. A new brand of sweets of varying colours is produced and the different colours occur in different proportions. The table below gives the probability that a randomly chosen sweet has each colour, but the value for tan candies is missing.

Colour	Brown	Red	Yellow	Green	Orange	Tan
Probability	0.3	0.2	0.2	0.1	0.1	?

3.2.

(a) What value must the missing probability be?

(b) You draw a sweet at random from a packet. What is the probability of each of the following events?

i. You get a brown one or a red one.
ii. You don't get a yellow one.
iii. You don't get either an orange one or a tan one.
iv. You get one that is brown or red or yellow or green or orange or tan.

3.2. 2% of the population have a certain blood disease in a serious form; 10% have it in a mild form; and 88% don't have it at all. A new blood test is developed; the probability of testing positive is $9/10$ if the subject has the serious form, $6/10$ if the subject has the mild form, and $1/10$ if the subject doesn't have the disease. I have just tested positive. What is the probability that I have the serious form of the disease? (3dp)